PISA 2012 RELEASED MATHEMATICS ITEMS

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FOREWORD

This document is a consolidated publication of the paper-based mathematics survey questions released after the 2012 administration of the OECD's Programme for International Student Assessment (PISA). It includes paper-based items from the 2012 survey administration, items developed for the 2012 survey that were used in the field trial that took place in 2011 but were not selected for use in the main survey in 2012, and items from the PISA 2003 survey that were also used in PISA 2006 but were then held in reserve and are released now for the first time.

The PISA survey questions are presented here in 'unit form'. PISA items in this form comprise a piece of stimulus, one or more questions related to that stimulus (with each question being referred to as an 'item') and, for each question, a set of guidelines that define the possible student response options and a proposed scoring scheme based on the defined response codes (the 'coding guide'). When students undertake the PISA assessment, they are presented with test booklets that contain only the stimulus and items from a sample of the available units that is determined by the test design applying to that assessment administration. The coding guide becomes relevant after students have responded to the test items, at the stage of coding and scoring the student responses. Consolidated coding guides for all items are published separately for use by trained item coders in each participating country.

<u>Section 1</u> contains the 26 items from two complete clusters of units that were part of the PISA 2012 survey design. The units named *Apartment purchase*, *Drip rate*, *Charts*, *Sailing ships*, *Sauce*, *Ferris wheel*, *Climbing Mount Fuji*, *Helen the cyclist*, *Which car?*, *Garage*, and *Revolving door* were selected for inclusion in the main survey test instruments for PISA 2012.

<u>Section 2</u> contains 30 items that had been developed for the PISA 2012 survey, but were not included in the final selection for the 2012 survey instruments. The units named *Memory* stick, Faulty players, Ice-cream shop, Oil spill, MP3 player, Penguins, Power of the wind, A construction with dice, Holiday apartment, DVD rental, Cable television, and Selling newspapers were in this group.

<u>Section 3</u> contains 11 items from a cluster of units developed for and used in the PISA 2003 survey and used again in PISA 2006, but which had been held in reserve since then (they were not needed in the PISA 2009 or 2012 instruments). They are from the units named *Car drive*, *Height, Making a booklet, Bicycles*, and *Seeing the tower*.

Ten items from three units used in the computer-based mathematics test that was an optional component of the PISA 2012 survey administration have also been released, and will made be available on line. These units are named *CD production, Star points*, and *Body mass index*.

The OECD holds the copyright for all PISA test items, and any questions about the use of the items should be referred to the PISA Secretariat at the OECD.

SECTION 1: PISA 2012 MAIN SURVEY ITEMS

APARTMENT PURCHASE

This is the plan of the apartment that George's parents want to purchase from a real estate agency.



Translation Note: In this unit please retain metric units throughout.

Translation Note: Translate the term "real estate agency" into local terminology for businesses that sell houses.

Question 1: APARTMENT PURCHASE

PM00FQ01-0 1 9

To estimate the total floor area of the apartment (including the terrace and the walls), you can measure the size of each room, calculate the area of each one and add all the areas together.

However, there is a more efficient method to estimate the total floor area where you only need to measure 4 lengths. Mark on the plan above the **four** lengths that are needed to estimate the total floor area of the apartment.

Translation Note: In some languages the term used for "area" varies according to the context. As this unit focuses on the areas of rooms, you may choose to use in the first instance here both terms with one between parentheses as in the FRE source version: "La superficie (l'aire) totale de l'appartement".

APARTMENT PURCHASE SCORING 1

QUESTION INTENT:

Description: Use spatial reasoning to show on a plan (or by some other method) the minimum number of side lengths needed to determine floor area Mathematical content area: Space and shape

Context: Personal

Process: Formulate

Full Credit

Code 1: Has indicated the four dimensions needed to estimate the floor area of the apartment on the plan. There are 9 possible solutions as shown in the diagrams below.



 A = (9.7m x 8.8m) – (2m x 4.4m), A = 76.56m² [Clearly used only 4 lengths to measure and calculate required area.]

No Credit

Code 0: Other responses.

DRIP RATE

Infusions (or intravenous drips) are used to deliver fluids and drugs to patients.



Nurses need to calculate the drip rate, D, in drops per minute for infusions.

They use the formula $D = \frac{dv}{60n}$ where

d is the drop factor measured in drops per millilitre (mL)

- v is the volume in mL of the infusion
- *n* is the number of hours the infusion is required to run.

Translation Note: Use relevant and appropriate words or expressions that are used for infusions (or intravenous drips) – there does not need to be two equivalent terms used if one is well known. Please also avoid using names of infusion brands (such as Baxter in French) even if they are well known.

Translation Note: Please use the appropriate scientific/medical term for "drop factor".

Translation Note: Please use consistently the national convention for writing the abbreviation of millilitre (mL or ml).

Translation Note: Initial letters of key words used in the formula may be adapted to suit a different language, but be careful that the changed letter does not conflict with letters used elsewhere in a unit or the formula.

Translation Note: Change expressions and symbols into the standard conventions for writing formulas in your country. For example, you may need to insert a sign like ' or * or x between the pronumerals to specify multiplication.

Question 1: DRIP RATE

A nurse wants to double the time an infusion runs for.

Describe precisely how *D* changes if *n* is **doubled** but *d* and *v* do not change.

DRIP RATE SCORING 1

QUESTION INTENT:

Description: Explain the effect that doubling one variable in a formula has on the resulting value if other variables are held constant Mathematical content area: Change and relationships Context: Occupational Process: Employ

Full Credit

Code 2: Explanation describes <u>both the direction of the effect and its size</u>.

- It halves
- It is half
- D will be 50% smaller
- D will be half as big

Partial Credit

- Code 1: A response which correctly states EITHER the direction OR the size of the effect, but not BOTH.
 - D gets smaller [no size]
 - There's a 50% change [no direction]
 - D gets bigger by 50%. [incorrect direction but correct size]

No Credit

- Code 0: Other responses.
 - D will also double [Both the size and direction are incorrect.]
- Code 9: Missing.

Question 3: DRIP RATE

Nurses also need to calculate the volume of the infusion, v, from the drip rate, D.

An infusion with a drip rate of 50 drops per minute has to be given to a patient for 3 hours. For this infusion the drop factor is 25 drops per millilitre.

What is the volume in mL of the infusion?

Volume of the infusion: mL

DRIP RATE SCORING 3

QUESTION INTENT:

Description: Transpose an equation and substitute two given values Mathematical content area: Change and relationships Context: Occupational Process: Employ

Full Credit

Code 1: 360 or a correctly transposed and substituted solution.

- 360
- + (60 × 3 × 50) \div 25 [Correct transposition and substitution.]

No Credit

Code 0: Other responses.

CHARTS

In January, the new CDs of the bands *4U2Rock* and *The Kicking Kangaroos* were released. In February, the CDs of the bands *No One's Darling* and *The Metalfolkies* followed. The following graph shows the sales of the bands' CDs from January to June.



Translation Note: The term "charts" does not refer to the mathematical term, but to the weekly listing of the best selling music CDs.

Translation Note: Translate band names with fictitious band names in your language.

Translation Note: The names of the months are shown in abbreviated form in the graphic. Full names can be used if space allows, as shown in the FRE version.

Question 1: CHARTS

PM918Q01

How many CDs did the band The Metalfolkies sell in April?

A 250

- B 500
- C 1000
- D 1270

CHARTS SCORING 1

QUESTION INTENT:

Description: Read a bar chart Mathematical content area: Uncertainty and data Context: Societal Process: Interpret

Full Credit

Code 1: B. 500

No Credit

Code 0: Other responses.

Code 9: Missing.

Question 2: CHARTS

PM918Q02

In which month did the band *No One's Darling* sell more CDs than the band *The Kicking Kangaroos* for the first time?

- A No month
- B March
- C April
- D May

CHARTS SCORING 2

QUESTION INTENT:

Description: Read a bar chart and compare the height of two bars Mathematical content area: Uncertainty and data Context: Societal Process: Interpret

Full Credit

Code 1: C. April

No Credit

Code 0: Other responses.

Question 5: CHARTS

The manager of *The Kicking Kangaroos* is worried because the number of their CDs that sold decreased from February to June.

What is the estimate of their sales volume for July if the same negative trend continues?

- A 70 CDs
- B 370 CDs
- C 670 CDs
- D 1340 CDs

CHARTS SCORING 5

QUESTION INTENT:

Description: Interpret a bar chart and estimate the number of CDs sold in the future assuming that the linear trend continues Mathematical content area: Uncertainty and data Context: Societal Process: Employ

Full Credit

Code 1: B. 370 CDs

No Credit

Code 0: Other responses.

SAILING SHIPS

Ninety-five percent of world trade is moved by sea, by roughly 50 000 tankers, bulk carriers and container ships. Most of these ships use diesel fuel.

Engineers are planning to develop wind power support for ships. Their proposal is to attach kite sails to ships and use the wind's power to help reduce diesel consumption and the fuel's impact on the environment.

Translation Note: "© by skysails": Do not adapt skysails as this is a registered label.

© by skysails

Question 1: SAILING SHIPS

PM923Q01

One advantage of using a kite sail is that it flies at a height of 150 m. There, the wind speed is approximately 25% higher than down on the deck of the ship.

At what approximate speed does the wind blow into a kite sail when a wind speed of 24 km/h is measured on the deck of the ship?

- A 6 km/h
- B 18 km/h
- C 25 km/h
- D 30 km/h
- E 49 km/h

Translation Note: In this unit please retain metric units throughout.

SAILING SHIPS SCORING 1

QUESTION INTENT:

Description: Apply calculation of percentage within a given real world situation Mathematical content area: Quantity Context: Scientific Process: Employ

Full Credit

Code 1: D. 30 km/h

No Credit

Code 0: Other responses.

Question 3: SAILING SHIPS

Approximately what is the length of the rope for the kite sail, in order to pull the ship at an angle of 45° and be at a vertical height of 150 m, as shown in the diagram opposite?

- A 173 m
- B 212 m
- C 285 m
- D 300 m

Rope 45° 90° 150 m 150 m 150 m Note: Drawing not to scale. © by skysails

SAILING SHIPS SCORING 3

QUESTION INTENT:

Description: Use Pythagorean Theorem within a real geometric context Mathematical content area: Space and shape Context: Scientific Process: Employ

Full Credit

Code 1: B. 212 m

No Credit

Code 0: Other responses.

Code 9: Missing.

PM923Q03

Question 4: SAILING SHIPS

Due to high diesel fuel costs of 0.42 zeds per litre, the owners of the ship *NewWave* are thinking about equipping their ship with a kite sail.

It is estimated that a kite sail like this has the potential to reduce the diesel consumption by about 20% overall.

Name: NewWaveType: freighterLength: 117 metresBreadth: 18 metresLoad capacity: 12 000 tonsMaximum speed: 19 knotsDiesel consumption per year without a kite sail: approximately 3 500 000 litres

The cost of equipping the *NewWave* with a kite sail is 2 500 000 zeds.

After about how many years would the diesel fuel savings cover the cost of the kite sail? Give calculations to support your answer.

Number of years:....

Translation Note: Change to , instead of . for decimal points, if that is your standard usage.

SAILING SHIPS SCORING 4

QUESTION INTENT:

Description: Solve a real world situation involving cost savings and fuel consumption

Mathematical content area: Change and relationships

Context: Scientific

Process: Formulate

Full Credit

Code 1: A solution from 8 to 9 years is provided with adequate (mathematical) calculations.

Diesel consumption per year without a sail: 3.5 million litres, price 0.42 zed/litre, costs for diesel without a sail 1 470 000 zeds. If 20% is saved with the sail this results in a saving of 1 470 000 x 0.2 = 294 000 zeds per year. Thus: 2 500 000/294 000 ≈ 8.5, i.e.: After about 8 to 9 years, the sail becomes (financially) worthwhile.

No Credit

- Code 0: Other responses.
- Code 9: Missing.

SAUCE

Question 2: SAUCE

PM924Q02 - 0 1 9

You are making your own dressing for a salad.

Here is a recipe for 100 millilitres (mL) of dressing.

Salad oil:	60 mL
Vinegar:	30 mL
Soy sauce:	10 mL

How many millilitres (mL) of salad oil do you need to make 150 mL of this dressing?

Answer: mL

Translation Note: In this unit please retain metric units throughout.

SAUCE SCORING 2

QUESTION INTENT:

Description: Apply ratio concept in daily life situation to calculate the amount of one ingredient required in a recipe Mathematical content area: Quantity Context: Personal Process: Formulate

Full Credit

Code 1: 90 • 60 + 30

No Credit

Code 0: Other responses. • 1.5 times more

FERRIS WHEEL

A giant Ferris wheel is on the bank of a river. See the picture and diagram below.



The Ferris wheel has an external diameter of 140 metres and its highest point is 150 metres above the bed of the river. It rotates in the direction shown by the arrows.

Translation Note: In this unit please retain metric units throughout.

Question 1: FERRIS WHEEL

PM934Q01-0 1 9

The letter *M* in the diagram indicates the centre of the wheel.

How many metres (m) above the bed of the river is point M?

Answer: m

FERRIS WHEEL SCORING 1

QUESTION INTENT:

Description: Calculate length based on information in a 2-D drawing Mathematical content area: Space and shape Context: Societal Process: Employ

Full Credit

Code 1: 80

No Credit

Code 0: Other responses.

Code 9: Missing.

Question 2: FERRIS WHEEL

The Ferris wheel rotates at a constant speed. The wheel makes one full rotation in exactly 40 minutes.

John starts his ride on the Ferris wheel at the boarding point, P.

Where will John be after half an hour?

- A At R
- B Between R and S
- C At S
- D Between S and P

FERRIS WHEEL SCORING 2

QUESTION INTENT:

Description: Estimate location based on the rotation of an object and specified time taken Mathematical content area: Space and shape Context: Societal Process: Formulate

Full Credit

Code 1: C. At S

No Credit

Code 0: Other responses.

Code 9: Missing.

PM934Q02

CLIMBING MOUNT FUJI

Mount Fuji is a famous dormant volcano in Japan.



Translation Note: Please do not change the names of locations or people in this unit: retain "Mount Fuji", "Gotemba" and "Toshi".

Question 1: CLIMBING MOUNT FUJI

PM942Q01

Mount Fuji is only open to the public for climbing from 1 July to 27 August each year. About 200 000 people climb Mount Fuji during this time.

On average, about how many people climb Mount Fuji each day?

- A 340
- B 710
- C 3400
- D 7100
- E 7400

CLIMBING MOUNT FUJI SCORING 1

QUESTION INTENT:

Description: Identify an average daily rate given a total number and a specific time period (dates provided) Mathematical content area: Quantity Context: Societal Process: Formulate

Full Credit

Code 1: C. 3400

No Credit

Code 0: Other responses.

Question 2: CLIMBING MOUNT FUJI

The Gotemba walking trail up Mount Fuji is about 9 kilometres (km) long.

Walkers need to return from the 18 km walk by 8 pm.

Toshi estimates that he can walk up the mountain at 1.5 kilometres per hour on average, and down at twice that speed. These speeds take into account meal breaks and rest times.

Using Toshi's estimated speeds, what is the latest time he can begin his walk so that he can return by 8 pm?

.....

Translation Note: Please use local convention for stating times of the day, and for writing decimal values with , instead of . .

Translation Note: In this unit please retain metric units throughout.

CLIMBING MOUNT FUJI SCORING 2

QUESTION INTENT:

Description: Calculate the start time for a trip given two different speeds, a total distance to travel and a finish time Mathematical content: Change and relationships Context: Societal Process: Formulate

Full Credit

Code 1: 11 (am) [with or without am, or an equivalent way of writing time, for example, 11:00]

No Credit

- Code 0: Other responses.
- Code 9: Missing.

Question 3: CLIMBING MOUNT FUJI

Toshi wore a pedometer to count his steps on his walk along the Gotemba trail.

His pedometer showed that he walked 22 500 steps on the way up.

Estimate Toshi's average step length for his walk up the 9 km Gotemba trail. Give your answer in centimetres (cm).

Answer: cm

CLIMBING MOUNT FUJI SCORING 3

QUESTION INTENT:

Description: Divide a length given in km by a specific number and express the quotient in cm Mathematical content: Quantity Context: Societal Process: Employ

Full Credit

Code 2: 40

Partial Credit

- Code 1: Responses with the digit 4 based on incorrect conversion to centimetres.
 - 0.4 [answer given in metres]
 - 4000 [incorrect conversion]

No Credit

- Code 0: Other responses.
- Code 9: Missing.

HELEN THE CYCLIST



Helen has just got a new bike. It has a speedometer which sits on the handlebar.

The speedometer can tell Helen the distance she travels and her average speed for a trip.

Question 1: HELEN THE CYCLIST

PM957Q01

On one trip, Helen rode 4 km in the first 10 minutes and then 2 km in the next 5 minutes.

Which one of the following statements is correct?

- A Helen's average speed was greater in the first 10 minutes than in the next 5 minutes.
- B Helen's average speed was the same in the first 10 minutes and in the next 5 minutes.
- C Helen's average speed was less in the first 10 minutes than in the next 5 minutes.
- D It is not possible to tell anything about Helen's average speed from the information given.

Translation Note: Throughout this Unit please retain metric units.

HELEN THE CYCLIST SCORING 1

QUESTION INTENT:

Description: Compare average speeds given distances travelled and times taken

Mathematical content area: Change and relationships

Context: Personal

Process: Employ

Full Credit

Code 1: B. Helen's average speed was the same in the first 10 minutes and in the next 5 minutes.

No Credit

- Code 0: Other responses.
- Code 9: Missing.

Question 2: HELEN THE CYCLIST

Helen rode 6 km to her aunt's house. Her speedometer showed that she had averaged 18 km/h for the whole trip.

Which one of the following statements is correct?

A It took Helen 20 minutes to get to her aunt's house.

B It took Helen 30 minutes to get to her aunt's house.

C It took Helen 3 hours to get to her aunt's house.

D It is not possible to tell how long it took Helen to get to her aunt's house.

HELEN THE CYCLIST SCORING 2

QUESTION INTENT:

Description: Calculate time travelled given average speed and distance travelled Mathematical content area: Change and relationships Context: Personal Process: Employ

Full Credit

Code 1: A. It took Helen 20 minutes to get to her aunt's house.

No Credit

Code 0: Other responses.

Code 9: Missing.

PM957Q02

Question 3: HELEN THE CYCLIST

Helen rode her bike from home to the river, which is 4 km away. It took her 9 minutes. She rode home using a shorter route of 3 km. This only took her 6 minutes.

What was Helen's average speed, in km/h, for the trip to the river and back?

Average speed for the trip:km/h

HELEN THE CYCLIST SCORING 3

QUESTION INTENT:

Description: Calculate average speed over two trips given two distances travelled and the times taken Mathematical content area: Change and relationships Context: Personal Process: Employ

Full Credit

Code 1: 28.

No Credit

Code 0: Other responses. • 28.3 [Incorrect method: average of speeds for 2 trips (26.67 and 30).]

WHICH CAR?

Chris has just received her car driving licence and wants to buy her first car.

This table below shows the details of four cars she finds at a local car dealer.



Model:	Alpha	Bolte	Castel	Dezal
Year	2003	2000	2001	1999
Advertised price (zeds)	4800	4450	4250	3990
Distance travelled (kilometres)	105 000	115 000	128 000	109 000
Engine capacity (litres)	1.79	1.796	1.82	1.783

Translation Note: Change the car's names to other more suitable fictional names if necessary – but keep the other numbers and values the same.

Translation Note: The use of zeds is important to the Unit, so please do not adapt "zed" into an existing currency.

Translation Note: Change to , instead of . for decimal points, if that is your standard usage, in EACH occurrence.

Question 1: WHICH CAR?

PM985Q01

Chris wants a car that meets **all** of these conditions:

- The distance travelled is **not** higher than 120 000 kilometres.
- It was made in the year 2000 or a later year.
- The advertised price is **not** higher than 4500 zeds.

Which car meets Chris's conditions?

- A Alpha
- B Bolte
- C Castel
- D Dezal

WHICH CAR? SCORING 1

QUESTION INTENT:

Description: Select a value that meets four numerical conditions/statements set

within a financial context Mathematical content area: Uncertainty and data Context: Personal Process: Interpret

Full Credit

Code 1: B Bolte.

No Credit

Code 0: Other responses.

Code 9: Missing.

Question 2: WHICH CAR?

PM985Q02

Which car's engine capacity is the smallest?

- A Alpha
- B Bolte
- C Castel
- D Dezal

WHICH CAR? SCORING 2

QUESTION INTENT:

Description: Choose the smallest decimal number in a set of four, in context Mathematical content area: Quantity Context: Personal Process: Employ

Full Credit

Code 1: D Dezal.

No Credit

Code 0: Other responses.

Question 3: WHICH CAR?

Chris will have to pay an extra 2.5% of the advertised cost of the car as taxes.

How much are the extra taxes for the Alpha?

Extra taxes in zeds:

WHICH CAR? SCORING 3

QUESTION INTENT:

Description: Calculate 2.5% of a value in the thousands within a financial context Mathematical content area: Quantity Context: Personal Process: Employ

Full Credit

Code 1: 120.

No Credit

Code 0: Other responses. • 2.5% of 4800 zeds [Needs to be evaluated.]

GARAGE

A garage manufacturer's "basic" range includes models with just one window and one door.

George chooses the following model from the "basic" range. The position of the window and the door are shown here.



Question 1: GARAGE

PM991Q01

The illustrations below show different "basic" models as viewed from the back. Only one of these illustrations matches the model above chosen by George.



Which model did George choose? Circle A, B, C or D.

GARAGE SCORING 1

QUESTION INTENT:

Description: Use space ability to identify a 3D view corresponding to another given 3D view Mathematical content area: Space and shape Context: Occupational Process: Interpret

Full Credit

Code 1: C [Graphic C]

No Credit

Code 0: Other responses.

Question 2: GARAGE

The two plans below show the dimensions, in metres, of the garage George chose.



Note: Drawing not to scale.

The roof is made up of two identical rectangular sections.

Calculate the total area of the roof. Show your work.



Translation Note: Decimal parts of measurements shown on diagram will need to be converted to use a , rather than a . as appropriate.

GARAGE SCORING 2

QUESTION INTENT:

Description: Interpret a plan and calculate the area of a rectangle using the Pythagorean theorem or measurement

Mathematical content area: Space and shape

Context: Occupational

Process: Employ

Full Credit

- Code 21: Any value from 31 to 33, either showing no working at all or supported by working that shows the use of the Pythagorean theorem (or including elements indicating that this method was used). [Units (m^2) not required]. • 12√7.25 m²

 - 12 × 2.69 = 32.28 m²
 - 32.4 m²

Partial Credit

- Code 11: Working shows correct use of the Pythagorean theorem but makes a calculation error or uses incorrect length or does not double roof area.
 - $2.5^2 + 1^2 = 6$, $12 \times \sqrt{6} = 29.39$ [correct use of Pythagoras theorem with calculation error]
 - $2^2 + 1^2 = 5$, $2 \times 6 \times \sqrt{5} = 26.8 \text{ m}^2$ [incorrect length used]
 - 6 × 2.6 = 15.6 [Did not double roof area.]
- Code 12: Working does not show use of Pythagorean theorem but uses reasonable value for width of roof (for example, any value from 2.6 to 3) and completes rest of calculation correctly.
 - 2.75 × 12 = 33
 - $\cdot 3 \times 6 \times 2 = 36$
 - $\cdot 12 \times 2.6 = 31.2$

No Credit

Code 00: Other responses.

- $2.5 \times 12 = 30$ [Estimate of width of roof lies outside the acceptable range which is from 2.6 to 3.]
- $3.5 \times 6 \times 2 = 42$ [Estimate of width of roof lies outside the acceptable range which is from 2.6 to 3.]

REVOLVING DOOR

A revolving door includes three wings which rotate within a circular-shaped space. The inside diameter of this space is 2 metres (200 centimetres). The three door wings divide the space into three equal sectors. The plan below shows the door wings in three different positions viewed from the top.



Translation Note: If the term for "wings" in the context of a revolving door is not familiar to 15-year olds in your country, you may wish to introduce the term as for example in the FRE source version: "Une porte à tambour est composée de trois « ailes », appelées vantaux, qui tournent au sein d'un espace circulaire."

Question 1: REVOLVING DOOR

PM995Q01-0 1 9

What is the size in degrees of the angle formed by two door wings?

Size of the angle:º

REVOLVING DOOR SCORING 1

QUESTION INTENT:

Description: Compute the central angle of a sector of a circle Mathematical content area: Space and shape Context: Scientific Process: Employ

Full Credit

Code 1: 120 [accept the equivalent reflex angle: 240].

No Credit

Code 0: Other responses.

Code 9: Missing.

Question 2: REVOLVING DOOR

The two door **openings** (the dotted arcs in the diagram) are the same size. If these openings are too wide the revolving wings cannot provide a sealed space and air could then flow freely between the entrance and the exit, causing unwanted heat loss or gain. This is shown in the diagram opposite.

What is the maximum arc length in centimetres (cm) that each door opening can have, so that air never flows freely between the entrance and the exit?

PM995Q02-0 1 9



Maximum arc length: cm

REVOLVING DOOR SCORING 2

QUESTION INTENT:

Description: Interpret a geometrical model of a real life situation to calculate the length of an arc Mathematical content area: Space and shape Context: Scientific Process: Formulate

Full Credit

Code 1: Answers in the range from 103 to 105. [Accept answers calculated as $1/6^{th}$ of the circumference $(\frac{100\pi}{3})$. Also accept an answer of 100 only if it is clear that this response resulted from using $\pi = 3$. Note: Answer of 100 without supporting working could be obtained by a simple guess that it is the same as the radius (length of a single wing).]

No Credit

- Code 0: Other responses.
 - 209 [states the total size of the openings rather than the size of "each" opening].
- Code 9: Missing.

Question 3: REVOLVING DOOR

The door makes 4 complete rotations in a minute. There is room for a maximum of two people in each of the three door sectors.

What is the maximum number of people that can enter the building through the door in 30 minutes?

- A 60
- B 180
- C 240
- D 720

REVOLVING DOOR SCORING 3

QUESTION INTENT:

Description: Identify information and construct an (implicit) quantitative model to solve the problem Mathematical content area: Quantity Context: Scientific Process: Formulate

Full Credit

Code 1: D. 720

No Credit

Code 0: Other responses.

SECTION 2: PISA 2012 FIELD TRIAL ITEMS

MEMORY STICK

A memory stick is a small, portable computer storage device.

Ivan has a memory stick that stores music and photos. The memory stick has a capacity of 1 GB (1000 MB). The graph below shows the current disk status of his memory stick.



Translation Note: Please translate "memory stick" with the commonly used term in your language, for example, "USB key".
Question 1: MEMORY STICK

Ivan wants to transfer a photo album of 350 MB onto his memory stick, but there is not enough free space on the memory stick. While he does not want to delete any existing photos, he is happy to delete up to two music albums.

Album	Size
Album 1	100 MB
Album 2	75 MB
Album 3	80 MB
Album 4	55 MB
Album 5	60 MB
Album 6	80 MB
Album 7	75 MB
Album 8	125 MB

Ivan's memory stick has the following size music albums stored on it.

By deleting at most two music albums is it possible for Ivan to have enough space on his memory stick to add the photo album? Circle "Yes" or "No" and show calculations to support your answer.

Answer: Yes / No

.....

MEMORY STICK SCORING 1

QUESTION INTENT:

Description: Compare and calculate values to satisfy given criteria Mathematical content area: Quantity Context: Personal Process: Interpret

Full Credit

Code 1: YES, explicitly or implicitly, AND give any example of a combination of two albums that use 198 MB of space or more.

- He needs to delete 198 MB (350-152) so he could erase any two music albums that added up to more than 198 MB, for example albums 1 and 8.
- Yes, he could delete Albums 7 and 8 which gives available space of 152 + 75 + 125 = 352 MB.

No Credit

- Code 0: Other responses.
- Code 9: Missing.

Question 2: MEMORY STICK

During the following weeks, Ivan deletes some photos and music, but also adds new files of photos and music. The new disk status is shown in the table below:

Music	550 MB
Photos	338 MB
Free space	112 MB

His brother gives him a new memory stick with a capacity of 2GB (2000 MB), which is totally empty. Ivan transfers the content of his old memory stick onto the new one.

Which one of the following graphs represents the new memory stick's disk status? Circle A, B, C or D.



MEMORY STICK SCORING 2

QUESTION INTENT:

Description: Understand the relationships between the language of a problem and the symbolic and formal language needed to represent it mathematically Mathematical content area: Uncertainty and data

Context: Personal

Process: Interpret

Full Credit

Code 1: D

No Credit

Code 0: Other responses.

FAULTY PLAYERS

The *Electrix Company* makes two types of electronic equipment: video and audio players. At the end of the daily production, the players are tested and those with faults are removed and sent for repair.

The following table shows the average number of players of each type that are made per day, and the average percentage of faulty players per day.

Player type	Average number of players made per day	Average percentage of faulty players per day
Video players	2000	5%
Audio players	6000	3%

Question 1: FAULTY PLAYERS

PM00EQ01

Below are three statements about the daily production at *Electrix Company*. Are the statements correct?

Circle "Yes" or "No" for each statement.

Statement	Is the statement correct?
One third of the players produced daily are video players.	Yes / No
In each batch of 100 video players made, exactly 5 will be faulty.	Yes / No
If an audio player is chosen at random from the daily production for testing, the probability that it will need to be repaired is 0.03.	Yes / No

FAULTY PLAYERS SCORING 1

QUESTION INTENT:

Description: Interpret statistical information involving uncertainty Mathematical content area: Uncertainty and data Context: Occupational Process: Formulate

Full Credit

Code 1: Three correct responses: No, No, Yes, in that order.

No Credit

Code 0: Other responses.

Code 9: Missing.

Question 2: FAULTY PLAYERS

PM00EQ02-0 1 9

One of the testers makes the following claim:

"On average, there are more video players sent for repair per day compared to the number of audio players sent for repair per day."

Decide whether or not the tester's claim is correct. Give a mathematical argument to support your answer.

FAULTY PLAYERS SCORING 2

QUESTION INTENT:

Description: Interpret statistical information involving uncertainty Mathematical content area: Uncertainty and data Context: Occupational Process: Interpret

Full Credit

Code 1: Adequate explanation for why the tester is not correct.

- The tester is not correct; 5% of 2000 is 100, but 3% of 6000 is 180. So on average 180 audio players are sent for repair, which is more than the average of 100 video players sent for repair.
- The tester is not correct; the fault rate of video players is 5%, which is a little less than twice as high as the fault rate of audio players. But they make 6000 audio players, which is three times the number of video players, so the actual number of audio players sent for repair will be higher.

No Credit

Code 0: Other responses.

Question 3: FAULTY PLAYERS

The *Tronics Company* also makes video and audio players. At the end of the daily production runs, the *Tronics Company's* players are tested and those with faults are removed and sent for repair.

The tables below compare the average number of players of each type that are made per day, and the average percentage of faulty players per day, for the two companies.

Company	Average number of <u>video</u> players made per day	Average percentage of faulty players per day
Electrix Company	2000	5%
Tronics Company	7000	4%

Company	Average number of <u>audio</u> players made per day	Average percentage of faulty players per day
Electrix Company	6000	3%
Tronics Company	1000	2%

Which of the two companies, *Electrix Company* or *Tronics Company*, has the lower overall percentage of faulty players? Show your calculations using the data in the tables above.

FAULTY PLAYERS SCORING 3

QUESTION INTENT:

Description: Interpret statistical information involving uncertainty Mathematical content area: Uncertainty and data Context: Occupational Process: Formulate

Full Credit

Code 1: An adequate mathematical explanation for choosing the Electrix Company.
The Electrix Company. Because 5% of 2000 is 100 and 3% of 6000 is 180, so on average 280 players of Electrix Company's daily production are sent for repair; 280 out of 8000 gives an overall fault rate of 3.5%. A similar calculation for Tronics Company shows they have an overall fault rate of 3.75%. [Percentage calculations must be shown for full credit.]

No Credit

- Code 0: Other responses.
- Code 9: Missing.

ICE-CREAM SHOP

This is the floor plan for Mari's Ice-cream Shop. She is renovating the shop.

The service area is surrounded by the serving counter.



Note: Each square on the grid represents 0.5 metres x 0.5 metres.

Question 1: ICE-CREAM SHOP

PM00LQ01-0 1 2 9

Mari wants to put new edging along the outer edge of the counter. What is the total length of edging she needs? Show your work.

ICE-CREAM SHOP SCORING 1

QUESTION INTENT:

Description: Use Pythagorean theorem or accurate use of measurement to find hypotenuse of a right triangle and convert measures on a scale drawing Mathematical content area: Space and shape Context: Occupational Process: Employ

Full Credit

Code 2: From 4.5 to 4.55. [m or metres – with or without units.]

Partial Credit

Code 1: Responses which indicate some correct working (such as use of Pythagoras or reading the scale) but with an error such as incorrect use of the scale or a miscalculation.

- From 9 to 9.1. [Did not use the scale.]
- 2.5 m (or 5 units). [Used Pythagoras to calculate hypotenuse of 5 units (2.5 metres) but did not add on the two straight sides.]

No Credit

Code 0: Other responses.

Code 9: Missing.

Question 2: ICE-CREAM SHOP

PM00LQ02-0 1 2 9

Mari is also going to put new flooring in the shop. What is the total floor space area of the shop, excluding the service area and counter? Show your work.

Translation Note: "The total floor space area": In some languages the term used for "area" varies according to the context. You may choose to use both terms in the first occurrence, with one between parentheses as in the FRE source version: "La superficie (l'aire) totale".

ICE-CREAM SHOP 2: SCORING

QUESTION INTENT:

Description: Calculate area for polygonal shapes Mathematical content area: Space and shape Context: Occupational Process: Employ

Full Credit

Code 2: 31.5. [With or without units.]

Partial Credit

- Code 1: Working that clearly shows some correct use of the grid to calculate the area but with incorrect use of the scale or an arithmetical error.
 - 126. [Response which indicates correct calculation of the area but did not use the scale to get the real value.]
 - 7.5 x 5 (=37.5) 3 x 2.5 (=7.5) ½ x 2 x 1.5 (=1.5) = 28.5 m². [Subtracted instead of adding the triangular area when breaking total area down into sub areas.]

No Credit

- Code 0: Other responses.
- Code 9: Missing.

Question 3: ICE-CREAM SHOP

PM00LQ03-0 1 9



Mari wants to have sets of tables and four chairs like the one shown above in her shop. The circle represents the floor space area needed for each set.

For customers to have enough room when they are seated, each set (as represented by the circle) should be placed according to the following constraints:

- Each set should be placed at least at 0.5 metres away from walls.
- Each set should be placed at least at 0.5 metres from other sets.

What is the maximum number of sets that Mari can fit into the shaded seating area in her shop?

Number of sets:

ICE-CREAM SHOP SCORING QUESTION 3

QUESTION INTENT:

Description: Use scale to and follow constraints to find the number of circles

that will fit into a polygonal shape Mathematical content area: Space and shape Context: Occupational Process: Employ

Full Credit

Code 1: 4.

No Credit

Code 0: Other responses.

OIL SPILL

An oil tanker at sea struck a rock, making a hole in the oil storage tanks. The tanker was about 65 km from land. After a number of days the oil had spread, as shown on the map below.



Translation Note: Please do not change the size of the image relative to the map scale. When printed, the scale length shown on the legend should equal 1.0 cm.

Translation Note: In this unit please retain metric units throughout.

Question 1: OIL SPILL

Using the map scale, estimate the area of the oil spill in square kilometres (km²).

Answer:km²

Translation Note: In some languages the term used for "area" varies according to the context. As this unit focuses on the area of the oil spill, you may choose to use in the first instance here both terms with one between parentheses as in the FRE source version: "La superficie (l'aire) de la nappe de pétrole".

OIL SPILL SCORING 1

QUESTION INTENT:

Description: Estimation of an irregular area on a map, using a given scale Mathematical content area: Space and shape Context: Scientific Process: Employ

Full Credit

Code 1: Answers in the range from 2200 to 3300.

No Credit

- Code 0: Other responses.
- Code 9: Missing.

MP3 PLAYERS



Translation Note: The use of zeds is important to the unit, so please do not adapt "zed" into an existing currency.

Question 2: MP3 PLAYERS

PM904Q02

Olivia added the prices for the MP3 player, the headphones and the speakers on her calculator.

The answer she got was 248.



Olivia's answer is incorrect. She made one of the following errors. Which error did she make?

- A She added one of the prices in twice.
- B She forgot to include one of the three prices.
- C She left off the last digit in one of the prices.
- D She subtracted one of the prices instead of adding it.

MP3 PLAYERS SCORING 2

QUESTION INTENT:

Description: Identify reason for error made in data entry for the addition of three monetary amounts on a calculator Mathematical content area: Quantity Context: Personal Process: Employ

Full Credit

Code 1: C. She left off the last digit in one of the prices.

No Credit

Code 0: Other responses.

Code 9: Missing.

Question 3: MP3 PLAYERS

PM904Q03

Music City has a sale. When you buy **two or more** items at the sale, Music City takes **20%** off the normal selling prices of these items.

Jason has 200 zeds to spend.

At the sale, what can he afford to buy?

Circle "Yes" or "No" for each of the following options.

Items	Can Jason buy the items with 200 zeds?
MP3 player and the headphones	Yes / No
MP3 player and the speakers	Yes / No
All 3 items – the MP3 player, the headphones and the speakers	Yes / No

MP3 PLAYERS SCORING 3

QUESTION INTENT:

Description: Decide whether a known monetary amount will be sufficient to purchase a selection of items at a given percentage discount Mathematical content area: Quantity Context: Personal Process: Interpret

Full Credit

Code 1: Three correct responses: Yes, Yes, No, in that order.

No Credit

- Code 0: Other responses.
- Code 9: Missing.

Question 4: MP3 PLAYERS

The normal selling price of the MP3 items **includes** a profit of 37.5%. The price without this profit is called the wholesale price.

The profit is calculated as a percentage of the wholesale price.

Do the formulae below show a correct relationship between wholesale price, *w*, and normal selling price, *s*?

Circle "Yes" or "No" for each of the following formulae.

Formulae	Is the formula correct?
s = w + 0.375	Yes / No
w = s - 0.375s	Yes / No
s = 1.375w	Yes / No
w = 0.625s	Yes / No

Translation Note: The names for "selling price" and "wholesale price" could be translated to normal local usage or applications, and similarly profit can be changed to a related concept for a margin or changed appropriately to be a government or local tax. The percent rate must not be changed from 37.5%, however. The formal writing of the algebraic symbols and formulae should be maintained but can be adapted to local conventions. The letters *s* and *w* can be adapted to the first letters of the translated words.

MP3 PLAYERS SCORING 4

QUESTION INTENT:

Description: Decide which algebraic formula correctly connects two monetary variables where one includes a fixed percentage margin Mathematical content area: Change and relationships Context: Occupational Process: Formulate

Full Credit

Code 1: Four correct responses: No, No, Yes, No, in that order.

No Credit

Code 0: Other responses.

PENGUINS



The animal photographer Jean Baptiste went on a year-long expedition and took numerous photos of penguins and their chicks.

He was particularly interested in the growth in the size of different penguin colonies.

Translation Note: In French, "penguin" is "manchot".

Question 1: PENGUINS

Normally, a penguin couple produces two eggs every year. Usually the chick from the larger of the two eggs is the only one that survives.

With rockhopper penguins, the first egg weighs approximately 78 g and the second egg weighs approximately 110 g.

By approximately how many percent is the second egg heavier than the first egg?

- A 29%
- B 32%
- C 41%
- D 71%

Translation Note: Rockhopper - Eudyptes chrysocome

UNIT PENGUINS SCORING 1

QUESTION INTENT:

Description: Calculate with percentage within a real context Mathematical content area: Quantity Context: Scientific Process: Employ

Full Credit

Code 1: C. 41%

No Credit

Code 0: Other responses.

Code 9: Missing.



PM921Q01

Question 2: PENGUINS

Jean wonders how the size of a penguin colony will change over the next few years. In order to determine this, he makes the following assumptions:

- At the beginning of the year, the colony consists of 10 000 penguins (5 000 couples).
- Each penguin couple raises one chick in the spring of each year.
- By the end of the year 20% of all the penguins (adults and chicks) will die.

At the end of the first year, how many penguins (adults and chicks) are there in the colony?

Number of penguins:

UNIT PENGUINS SCORING 2

QUESTION INTENT:

Description: Understand a real situation to calculate a concrete number based on change including percentage increase/decrease Mathematical content area: Quantity Context: Scientific Process: Formulate

Full Credit

Code 1: 12 000

No Credit

- Code 0: Other responses.
- Code 9: Missing.

Question 3: PENGUINS

Jean assumes the colony will continue to grow in the following manner:

- At the beginning of each year, the colony consists of equal numbers of male and female penguins who form couples.
- Each penguin couple raises one chick in the spring of each year.
- By the end of each year 20% of all the penguins (adults and chicks) will die.
- One year old penguins will also raise chicks.

Based on the above assumptions, which of the following formulae describes the total number of penguins, *P*, after 7 years?

- A $P = 10\ 000\ x\ (1.5\ x\ 0.2)^7$
- B $P = 10\ 000\ \text{x}\ (1.5\ \text{x}\ 0.8)^7$
- C $P = 10\ 000\ \text{x}\ (1.2\ \text{x}\ 0.2)^7$
- D $P = 10\,000 \,\mathrm{x} \,(1.2 \,\mathrm{x} \,0.8)^7$

UNIT PENGUINS SCORING 3

QUESTION INTENT:

Description: Understand a given situation and choose an appropriate mathematical model Mathematical content area: Change and relationships Context: Scientific Process: Formulate

Full Credit

Code 1: B. $P = 10\ 000\ x\ (1.5\ x\ 0.8)^7$

No Credit

Code 0: Other responses.

Question 4: PENGUINS

After he gets home from his trip, Jean Baptiste has a look on the Internet to see how many chicks a penguin couple raise on average.

He finds the following bar chart for the three penguin types Gentoo, Rockhopper and Magellanic.



Based on the chart above, are the following statements about these three penguin types true or false?

Circle "True" or "False" for each statement.

Statement	Is the statement true or false?
In 2000, the average number of chicks raised per penguin couple was larger than 0.6.	True / False
In 2006, on average, less than 80% of penguin couples raised a chick.	True / False
By about 2015 these three penguin types will be extinct.	True / False
The average number of Magellanic penguin chicks raised per penguin couple decreased between 2001 and 2004.	True / False

Translation Note: Gentoo - Pygoscelis papua / Rockhopper - Eudyptes chrysocome / Magellanic - Spheniscus magellanicus

Translation Note: Please translate "bar chart" with the term most commonly used in 15year olds' mathematics classes. Avoid using more formal expressions such as "histogram" in ENG or "histogramme" in FRE that are less common and more difficult for 15-year olds.

UNIT PENGUINS SCORING 4

QUESTION INTENT:

Description: Analyse different statements concerning a given bar chart Mathematical content area: Uncertainty and data Context: Scientific Process: Interpret

Full Credit

Code 1: Four correct responses: True, True, False, True in that order.

No Credit

Code 0: Other responses.



POWER OF THE WIND

Zedtown is considering building some wind power stations to produce electricity.

The Zedtown Council gathered information about the following model.

Model: Height of tower: Number of rotor blades: Length of one rotor blade: Maximum speed of rotation: Price for construction: Turnover: Maintenance cost: Efficiency: E-82 138 metres 3 40 metres 20 rotations per minute 3 200 000 zeds 0.10 zeds per kWh generated 0.01 zeds per kWh generated Operational 97% of the year

Note: kilowatt hours (kWh) is a measure of electrical energy.

Translation Note: In this unit please retain metric units throughout.

Translation Note: Change to , instead of . for decimal points, if that is your standard usage.

Question 1: POWER OF THE WIND

PM922Q01

Decide whether the following statements about the E-82 wind power station can be deduced from the information provided. Circle "Yes" or "No" for each statement.

Statement	Can this statement be deduced from the information provided?
The construction of three of the power stations will cost more than 8 000 000 zeds in total.	Yes / No
The maintenance costs for the power station correspond to approximately 5% of its turnover.	Yes / No
The maintenance costs for the wind power station depend on the amount of kWh generated.	Yes / No
On exactly 97 days a year, the wind power station is not operational.	Yes / No

POWER OF THE WIND SCORING 1

QUESTION INTENT:

Description: Analyse different information about a given scenario Mathematical content area: Change and relationships Context: Scientific Process: Interpret

Full Credit

Code 1: Four correct responses: Yes, No, Yes, No, in that order.

No Credit

Code 0: Other responses.

Code 9: Missing.

Question 2: POWER OF THE WIND

PM922Q02

Zedtown wants to estimate the costs and the profit that would be created by constructing this wind power station.

Zedtown's mayor proposes the following formula for estimating the financial gain, F zeds, over a number of years, y, if they build the E-82 model.

$$F = 400\ 000\ y - 3\ 200\ 000$$



Translation Note: Change expressions and symbols into the standard conventions for writing equations in your country.

Translation Note: Initial letters of key words used in the formula may be adapted to suit a different language, but be careful that the changed letter does not conflict with letters used elsewhere in the unit or the formula.

Based on the mayor's formula, what is the minimum number of years of operation required to cover the cost of construction of the wind power station?

- A 6 years
- B 8 years
- C 10 years
- D 12 years

POWER OF THE WIND SCORING 2

QUESTION INTENT:

Description: Understand and solve a given equation in a context Mathematical content area: Change and relationships Context: Scientific Process: Employ

Full Credit

Code 1: B. 8 years

No Credit

Code 0: Other responses.

Question 3: POWER OF THE WIND

Zedtown has decided to erect some E-82 wind power stations in a square field (length = breadth = 500 m).

According to building regulations, the minimum distance between the towers of two wind power stations of this model has to be five times the length of a rotor blade.

The town mayor has made a suggestion for how to arrange the wind power stations in the field. This is shown in the diagram opposite.

Explain why the town mayor's suggestion does not meet the building regulations. Support your arguments with calculations.



• = wind power station tower Note: Drawing is not to scale.

UNIT POWER OF THE WIND SCORING 3

QUESTION INTENT:

Description: Use Pythagorean Theorem within a real context Mathematical content area: Space and shape Context: Scientific Process: Employ

Full Credit

- Code 1: Response which shows correctly and comprehensibly in a mathematical way that the required minimum distance of five rotor blade lengths (i.e. 200 m) has not been adhered to between all the wind power stations. A sketch would be desirable but it is not imperative, just as a separate sentence containing the answer is not.
 - The wind power stations cannot be erected like this because their distance is sometimes only J125²+125² ≈ 177 m apart.

No Credit

- Code 0: Other responses.
- Code 9: Missing.

PM922Q03-0 1 9

Question 4: POWER OF THE WIND

What is the maximum speed that the ends of the rotor blades for the wind power station move? Describe your solution process and give the result in **kilometres per hour** (km/h). Refer back to the information about the E-82 model.

Maximum speed: km/h

POWER OF THE WIND SCORING 4

QUESTION INTENT:

Description: Use multistep modelling to solve a problem within a kinetic context Mathematical content area: Change and relationships Context: Scientific Process: Employ

Full Credit

- Code 2: The correct result is deduced from a correct, complete, and comprehensible solution process. The result has to be provided in km/h. A sketch is not imperative, just as a separate sentence containing the answer is not.
 - Maximum rotational speed is 20 rotations per minute; the distance per rotation is $2 \cdot \pi \cdot 40 \text{ m} \approx 250 \text{m}$; i.e. $20 \cdot 250 \text{ m/min} \approx 5000 \text{ m/min} \approx 83 \text{ m/s} \approx 300 \text{ km/h}$.

Partial Credit

- Code 1: The correct result is deduced from a correct, complete, and comprehensible solution process. However, the result is not provided in km/h. Here again, a sketch is not imperative, just as a separate sentence containing the answer is not.
 - Maximum rotational speed is 20 rotations per minute; the distance per rotation is $2 \cdot \pi \cdot 40 \text{ m} \approx 250 \text{ m}$; i.e. $20 \cdot 250 \text{ m/min} \approx 5000 \text{ m/min} \approx 83 \text{ m/s}$.

No Credit

- Code 0: Other responses.
- Code 9: Missing.

A CONSTRUCTION WITH DICE (1 ITEM)

In the picture below a construction has been made using seven identical dice with their faces numbered from 1 to 6.



When the construction is viewed from the top, only 5 dice can be seen.

Question 1: A CONSTRUCTION WITH DICE

PM937Q01-0 1 2 9

How many dots in total can be seen when this construction is viewed from the top?

Number of dots seen:

A CONSTRUCTION WITH DICE SCORING 1

QUESTION INTENT:

Description: Interpret a required perspective from the photo of a 3 dimensional construction Mathematical content area: Space and shape Context: Personal Process: Interpret

Full Credit

Code 2: 17

Partial Credit

Code 1: 16

No Credit

Code 0: Other responses.

HOLIDAY APARTMENT

Christina finds this holiday apartment for sale on the internet. She is thinking about buying the holiday apartment so that she can rent it out to holiday guests.

Number of rooms:	1 x living and dining room 1 x bedroom 1 x bathroom	43
Size:	60 square metres (m ²)	
Parking spot:	yes	
Travel time to town centre:	10 minutes	
Distance to the beach:	350 metres (m) in a direct line	1
Average usage by holiday guests in the last 10 years:	315 days per year	1



Question 1: HOLIDAY APARTMENT

To assess the price of the holiday apartment, Christina has asked for an expert's evaluation. To estimate the value of a holiday apartment, the expert uses the following criteria:

Price per m ²	Base price:	2500 zeds per m²			
Additional value criteria	Travel time to town centre:	More than 15 minutes: +0 zeds	From 5 to 15 minutes: +10 000 zeds	Less than 5 minutes: +20 000 zeds	
	Distance to beach (in a direct line):	More than 2 km: +0 zeds	From 1 to 2 km: +5000 zeds	From 0.5 to 1 km: +10 000 zeds	Less than 0.5 km: +15 000 zeds
	Parking spot:	No: +0 zeds	Yes: +35 000 zeds		

If the value estimated by the expert is greater than the advertised selling price, the price is considered to be "very good" for Christina as the potential buyer.

Show that based on the expert's criteria, the selling price on offer is "very good" for Christina.

HOLIDAY APARTMENT SCORING 1

QUESTION INTENT:

Description: Evaluate a number of criteria against the advertised selling price of a holiday apartment

Mathematical content area: Quantity

Context: Societal

Process: Employ

Full Credit

Code 1: A response that shows that the estimated value according to the expert's criteria is 210 000 zeds which is more than 200 000 zeds hence making it a "very good" price. [The expert's value of 210 000 zeds must be explicitly stated, but the advertised price can be referred to implicitly or explicitly].

- The expert's total is 210 000 zeds which is greater than the advertised price of 200 000 which means it is a very good price.
- The total of 210 000 zeds is greater than the advertised price.

No Credit

Code 0: Other responses.

Code 9: Missing.

Question 2: HOLIDAY APARTMENT

PM962Q02

315 days per year is the average usage of the apartment by holiday guests over the last 10 years.

Decide whether the following statements can be deduced from this information. Circle "Yes" or "No" for each statement.

Statement	Can the statement be deduced from the given data?
It can be said with certainty that the holiday apartment was used on exactly 315 days by holiday guests in at least one of the last 10 years.	Yes / No
Theoretically it is possible that in the last 10 years the apartment was used on more than 315 days every year by holiday guests.	Yes / No
Theoretically it is possible that in one of the last 10 years the apartment was not used at all by holiday guests.	Yes / No

Note: Assume a year has 365 days.

HOLIDAY APARTMENT SCORING 2

QUESTION INTENT:

Description: Interpret the meaning of a given average value Mathematical content area: Uncertainty and data Context: Societal Process: Interpret

Full Credit

Code 1: Three correct responses: No, No, Yes, in that order.

No Credit

Code 0: Other responses.

DVD RENTAL

Jenn works at a store that rents DVDs and computer games.

At this store the annual membership fee costs 10 zeds.

The DVD rental fee for members is lower than the fee for nonmembers, as shown in the following table:

Non-member rental	Member rental fee	
fee for one DVD	for one DVD	
3.20 zeds	2.50 zeds	



Translation Note: Change to , instead of . for decimal points, if that is your standard usage, in EACH occurrence.

Translation Note: The use of zeds is important to the Unit, so please do not adapt "zed" into an existing currency.

Question 1: DVD RENTAL

PM977Q01-0 1 9

Troy was a member of the DVD rental store last year.

Last year he spent 52.50 zeds in total, which included his membership fee.

How much would Troy have spent if he had not been a member but had rented the same number of DVDs?

Number of zeds:

DVD RENTAL SCORING 1

QUESTION INTENT:

Description: Calculate and compare numbers in an everyday situation Mathematical content area: Quantity Context: Personal Process: Employ

Full Credit

Code 1: 54.40.

No Credit

Code 0: Other responses.

Code 9: Missing.

Question 2: DVD RENTAL

PM977Q02-00 11 12 21 22 23 24 99

What is the minimum number of DVDs a member needs to rent so as to cover the cost of the membership fee? Show your work.

Number of DVDs:

DVD RENTAL SCORING 2

QUESTION INTENT:

Description: Calculate and compare numbers in an everyday situation Mathematical content area: Quantity Context: Personal Process: Formulate

Full Credit

Code 21: 15. [Algebraic solution with correct reasoning].

- 3.20x = 2.50x + 100.70x = 10
 - x = 10 / 0.70 = 14.2 approximately
 - but whole number solution is required: 15 DVDs
- 3.20x > 2.50x + 10 [Same steps as previous solution but worked as an inequality].
- Code 22: 15. [Arithmetical solution with correct reasoning].
 - For a single DVD, a member saves 0.70 zeds. Because a member has already paid 10 zeds at the beginning, they should at least save this amount for the membership to be worthwhile. 10 / 0.70 = 14.2... So 15 DVDs.
- Code 23: 15. [Solve correctly using systematic trial and error, where student chooses a number and finds the fee for members and non-members, and uses this to locate the correct number (15) for which a member pays less than a non-member].
 - 10 DVDs = 32 zeds non-members and 25 zeds + 10 zeds = 35 zeds for members.

Therefore try a higher number than 10. 15 DVDs is 54 zeds for non-members and 37.50 + 10 = 47.50 zeds for members.

Therefore try a smaller value: 14 DVDs = 44.80 zeds for non-members and 35

+10 = 45 zeds for members. Therefore 15 DVDs is the answer.

Code 24: 15. With other correct reasoning.

Partial Credit

Code 11: 15. No reasoning or working.

- Code 12: Correct calculation but with incorrect rounding or no rounding to take into account context.
 - 14
 - 14.2
 - 14.3
 - 14.28 ...

No Credit

Code 00: Other responses.
CABLE TELEVISION

The table below shows data about household ownership of televisions (TVs) for five countries.

It also shows the percentage of those households that own TVs and also subscribe to cable TV.



Country	Number of households that own TVs	Percentage of households that own TVs compared to all households	Percentage of households that subscribe to cable television compared to households that own TVs
Japan	48.0 million	99.8%	51.4%
France	24.5 million	97.0%	15.4%
Belgium	4.4 million	99.0%	91.7%
Switzerland	2.8 million	85.8%	98.0%
Norway	2.0 million	97.2%	42.7%

Source: ITU, World Telecommunication Indicators 2004/2005 ITU, World Telecommunication/ICT Development Report 2006

Translation Note: Please do not change the countries in this unit.

Translation Note: Change to , instead of . for decimal points, if that is your standard usage, in EACH occurrence.

Translation Note: You may change the term "cable TV" to a relevant local terminology, for example, "subscription TV" or "pay per view TV".

Translation Note: There may be no word for "million" in some languages; translate one million appropriately (e.g. ten hundred thousand); if absolutely necessary, the numeral 1 000 000 could be used throughout.

Question 1: CABLE TELEVISION

The table shows that in Switzerland 85.8% of all households own TVs.

Based on the information in the table, what is the closest estimate of the total number of households in Switzerland?

- A 2.4 million
- B 2.9 million
- C 3.3 million
- D 3.8 million

CABLE TELEVISION SCORING 1

QUESTION INTENT:

Description: Apply proportionality based on a set of data Mathematical content area: Uncertainty and data Context: Societal Process: Interpret

Full Credit

Code 1: C. 3.3 million.

No Credit

Code 0: Other responses.

Question 2: CABLE TELEVISION

Kevin looks at the information in the table for France and Norway.

Kevin says: "Because the percentage of all households that own TVs is almost the same for both countries, Norway has more households that subscribe to cable TV."

Explain why this statement is incorrect. Give a reason for your answer.

.....

CABLE TELEVISION SCORING 2

QUESTION INTENT:

Description: Understand proportionality based on data provided in a table Mathematical content area: Uncertainty and data Context: Societal Process: Interpret

Full Credit

Code 11: A response that says that Kevin needed to take into account the actual number of households with TVs for the two countries. [Accept "population" as a substitute for "households'].

- He is wrong because there are over 22 million more households that own TVs in France.
- Because the population of France is about 10 times more than Norway.
- Because France has more people, the number of people that have a TV is much larger, so the number of cable TV subscriptions is larger.

Code 12: A response that is based on calculation of the actual number of subscribers in the two countries.

• Because France has $(24.5 \times 0.154) =$ approximately 3.7 million households that subscribe to cable TV, while Norway has (2.0×0.427) which is approximately 0.8 million households. France has more cable television subscribers.

No Credit

Code 00: Other responses.

SELLING NEWSPAPERS

In Zedland there are two newspapers that try to recruit sellers. The posters below show how they pay their sellers.

ZEDLAND STAR
NEED EXTRA MONEY?
SELL OUR NEWSPAPER
You will be paid: 0.20 zeds per newspaper for the first 240 papers you sell in a week, plus 0.40 zeds for each additional

newspaper you sell.

ZEDLAND DAILY

WELL PAID JOB THAT TAKES LITTLE TIME!

Sell the *Zedland Daily* and make 60 zeds a week, plus an additional 0.05 zeds per newspaper you sell.

Translation Note: Change to , instead of . for decimal points, if that is your standard usage, in EACH occurrence.

Translation Note: The use of zeds is important to the unit, so please do not adapt "zed" into an existing currency.

Question 1: SELLING NEWSPAPERS

On average, Frederic sells 350 copies of the Zedland Star every week.

How much does he earn each week, on average?

Amount in zeds:

SELLING NEWSPAPERS SCORING 1

QUESTION INTENT:

Description: Identify relevant information for a simple mathematical model to calculate a number Mathematical content area: Change and relationships Context: Occupational Process: Formulate

PM994Q01-0 1 9

Full Credit

Code 1: 92 or 92.00.

No Credit

Code 0: Other responses.

Code 9: Missing.

Question 2: SELLING NEWSPAPERS

PM994Q02 - 0 1 9

Christine sells the Zedland Daily. One week she earns 74 zeds.

How many newspapers did she sell that week?

Number of newspapers sold:

SELLING NEWSPAPERS SCORING 2

QUESTION INTENT:

Description: Identify relevant information and transform this into a simple mathematical model to calculate a number Mathematical content area: Change and relationships Context: Occupational Process: Formulate

Full Credit

Code 1: 280.

No Credit

Code 0: Other responses.

Question 3: SELLING NEWSPAPERS

John decides to apply for a newspaper seller position. He needs to choose the *Zedland Star* or the *Zedland Daily*.

Which one of the following graphs is a correct representation of how the two newspapers pay their sellers? Circle A, B, C or D.



SELLING NEWSPAPERS SCORING 3

QUESTION INTENT:

Description: Identify correct mathematical models when two linear relationships are transformed to graphical representations Mathematical content area: Change and relationships Context: Occupational

Process: Interpret

Full Credit

Code 1: Graph C.

Code 0: Other responses.

SECTION 3: PISA 2006 ITEMS

CAR DRIVE

Kelly went for a drive in her car. During the drive, a cat ran in front of the car. Kelly slammed on the brakes and missed the cat.

Slightly shaken, Kelly decided to return home.

The graph below is a simplified record of the car's speed during the drive.



Question 1: CAR DRIVE

M302Q01

What was the maximum speed of the car during the drive?

Maximum speed:km/h.

CAR DRIVE SCORING 1

Full Credit

Code 1: 60 km/h.

No Credit

Code 0: Other responses.

Question 2: CAR DRIVE

What time was it when Kelly slammed on the brakes to avoid the cat?

Answer:

CAR DRIVE SCORING 2

Full Credit

Code 1: 9:06

OR

Six minutes past nine.

No Credit

Code 0: Other responses.

Code 9: Missing.

Question 3: CAR DRIVE

M302Q03-0 1 9

Was the route Kelly took to return home shorter than the distance she had travelled from home to the place where the incident with the cat occurred? Give an explanation to support your answer, using information given in the graph.

.....

.....

.....

CAR DRIVE SCORING 3

Full Credit

Code 1: Answer that the route home was shorter, with an adequate explanation. Explanation refers to BOTH lower average speed AND (approximately) equal time on return journey, or equivalent argument. Note that an argument based on the lower area under the graph for the return journey would also be given full credit.

The first part was longer than the trip home – it took the same amount of time but in the first part she was going much faster than the second part.

The route Kelly took home was shorter because it took her less time and she was travelling slower.

Code 0: Correct answer without adequate explanation.

It was shorter because when she slammed on the brakes she had just gone over half time.

The route home was shorter. It only covered 8 squares while the route there covered 9 squares.

OR

Other responses.

No it was the same because it took her six minutes to get back but she drove slower.

Looking at the graph if you include the time Kelly took to slow down for the cat it may have been a couple of seconds quicker but rounding off it was the same.

You can tell by the graph that it was the same distance to where she stopped as the distance back home.

HEIGHT

There are 25 girls in a class. The average height of the girls is 130 cm.

Question 1: HEIGHT

M421Q01-0 1 9

Explain how the average height is calculated.

HEIGHT SCORING 1

Full Credit

Code 1: Explanations that include: Sum the individual heights and divide by 25.
You add together every girl's height and divide by the number of girls.
Take all the girls' heights, add them up, and divide by the amount of girls, in this case 25.
The sum of all heights in the same unit divided by the number of girls.

No Credit

Code 0: Other responses.

Code 9: Missing.

Question 2: HEIGHT

M421Q02

Circle either "True" or "False" for each of the following statements.

Statement	True or False
If there is a girl of height 132 cm in the class, there must be a girl of height 128 cm.	True / False
The majority of the girls must have height 130 cm.	True / False
If you rank all of the girls from the shortest to the tallest, then the middle one must have a height equal to 130 cm.	True / False
Half of the girls in the class must be below 130 cm, and half of the girls must be above 130 cm.	True / False

HEIGHT SCORING 2

Full Credit

Code 1: False, False, False, False.

Code 0: Other responses.

Code 9: Missing.

Question 3: HEIGHT

M421Q03

An error was found in one student's height. It should have been 120 cm instead of 145 cm. What is the corrected average height of the girls in the class?

A 126 cm

B 127 cm

C 128 cm

D 129 cm

E 144 cm

HEIGHT SCORING 3

Full Credit

Code 1: D. 129 cm.

No Credit

Code 0: Other responses.

MAKING A BOOKLET

Question 1: MAKING A BOOKLET

M598Q01 - 0 1 9

Figure 1



Figure 1 shows how to make a small booklet. The instructions are given below:

- Take a piece of paper and fold it twice.
- Staple edge a.
- Cut open two edges at b.

The result is a small booklet with eight pages.



Figure 2 shows one side of a piece of paper that is used to make such a booklet. The page numbers have been put on the paper in advance.

The thick line indicates where the paper will be cut after folding.

Write the numbers 1, 4, 5 and 8 in the correct boxes in the following diagram to show which page number is directly behind each of the page numbers 2, 3, 6 and 7.



MAKING A BOOKLET SCORING 1

Full Credit

Code 1: Page numbers placed correctly in the following positions (ignore the orientation of the numbers):



No Credit

Code 0: Other responses.

BICYCLES

Justin, Samantha and Peter ride bicycles of different sizes. The following table shows the distance their bicycles travel for each complete turn of the wheels.

	Distance travelled in cm					
	1 turn	2 turns	3 turns	4 turns	5 turns	6 turns
Peter	96	192	288	384	480	
Samantha	160	320	480	640	800	
Justin	190	380	570	760	950	

Question 1: BICYCLES

M810Q01

Peter pushed his bike for three complete turns of his wheel. If Justin did the same with his bike, how much further would Justin's bike travel than Peter's? Give your answer in centimetres.

Answer:cm.

BICYCLES SCORING 1

Full Credit

Code 1: 282 cm.

No Credit

Code 0: Other responses.

Code 9: Missing.

Question 2: BICYCLES

M810Q02

How many turns of the wheel does it take for Samantha's bike to travel 1280 cm?

Answer:turns.

BICYCLES SCORING 2

Full Credit

Code 1: 8.

No Credit

Code 0: Other responses.

Code 9: Missing.

Question 3: BICYCLES

M810Q03-00 11 12 21 99

Peter's bicycle has a wheel circumference of 96 cm (or 0.96 m). It is a three-speed bicycle with a low, a middle and a high gear. The gear ratios of Peter's bicycle are:

Low 3:1 Middle 6:5 High 1:2

How many pedal turns would Peter take to travel 960 m in middle gear? Show your work.

NOTE: A gear ratio of 3:1 means 3 complete pedal turns yields 1 complete wheel turn.

BICYCLES SCORING 3

Full Credit

Code 21: 1200 pedal turns, with a fully correct method. Note that the correct answer, even when no working is shown, implies a fully correct method, and should be given full credit.

960 m requires 1000 wheel turns, which corresponds to $1000 \times \frac{6}{5} = 1200$ pedal

turns

Partial Credit

Code 11: 12 pedal turns, calculated by a correct method but without the correct conversion of units.

960 m requires 10 wheel turns (student forgot that distance in the table is given

in cm), which corresponds to $10 \times \frac{6}{5} = 12$ pedal turns

Code 12: Correct method but with other slight computational error or incomplete calculation.

3 pedal turns give 2.5 wheel turns, and 1 wheel turn = 0.96 metres, so 3 pedal turns = 2.4 metres. Therefore 960 m takes 400 pedal turns.

- 1000 wheel turns are needed (960/0.96) to travel 960 m, so 833 pedal turns are required in middle gear (5/6 of 1000). [The method is correct, but the ratio has been inverted.]
- 5 x 0.96 = 4.8, and 960/4.8 = 200, so 200 turns. Now 200/5 = 40 and
 40 x 6 = 240. So, 240 pedal turns are required. [A single error, the redundant first multiplication by 5, but an otherwise correct method.]

Code 00: Other responses.

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96000/5 = 19200, and 19200 × 6 = 115200 pedal turns. [The wheel circumference has not been taken into account.]
```

Code 99: Missing.

Translation Note: Beware of the different terms in French and English for the gear mechanism. 'Low gear', 'middle gear' and 'high gear' in English correspond respectively to 'grand pignon', 'pignon moyen' and 'petit pignon' in French. French focuses on the size of the gear wheel, while English focuses on the effect produced. With a large gear wheel ('grand pignon') one needs 3 pedal turns to obtain 1 wheel turn, thus the speed is lower ('low gear').

SEEING THE TOWER

Question 1: SEEING THE TOWER

In Figures 1 and 2 below, you see two drawings of the **same** tower. In Figure 1 you see **three** faces of the roof of the tower. In Figure 2 you see **four** faces.





In the following diagram, the view of the roof of the tower, from above, is shown. Five positions are shown on the diagram. Each is marked with a cross (\times) and they are labelled P1 – P5.

From each of these positions, a person viewing the tower would be able to see a number of faces of the roof of the tower.



In the table below, circle the number of faces that could be seen from each of these positions.

Position	Number of faces that could be seen from that position					
	(circle the correct number)					
P1	1	2	3	4	more than 4	
P2	1	2	3	4	more than 4	
P3	1	2	3	4	more than 4	
P4	1	2	3	4	more than 4	
P5	1	2	3	4	more than 4	

SEEING THE TOWER SCORING 1

Full Credit

Code 1: The following responses are circled for points P1-P5 respectively: 4, 3, 1, 2, 2

No Credit

- Code 0: Other responses.
- Code 9: Missing.

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